Microwave Imaging via Adaptive Beamforming Methods for Breast Cancer Detection

Bin Guo[1], Yanwei Wang[1], Jian Li[1], Petre Stoica[2], and Renbiao Wu[3]

[1] University of Florida, U. S. A.
[2] Uppsala University, Sweden
[3] Civil Aviation University of China, China

Abstract

Ultra-wideband (UWB) Microwave imaging (MWI) is a promising breast cancer detection technology which exploits the significant contrast in dielectric properties between normal breast tissue and tumor. Previously, data-independent methods, such as delay-and-sum (DAS) and space-time (ST) beamforming, have been used for microwave imaging. However, the low resolution and the poor interference suppression capability associated with the data-independent methods restrict their use in practice, especially when the noise is high and the backscattered signals are weak. In this paper, we develop two data-adaptive methods for microwave imaging, which are referred to as the robust weighted Capon beamforming (RWCB) method and the amplitude and phase estimation (APES) method. Due to their data- adaptive nature, these methods outperform their data-independent counterparts in terms of improved resolution and reduced sidelobe levels.

Introduction

Breast cancer is the most common nonskin malignancy in women and the second leading cause of female cancer mortality [1]. Early diagnosis is the key to survive from breast cancer [2]. As a safe, comfortable, sensitive, and accurate method, ultra-wideband (UWB) microwave imaging (MWI) is an attractive technology for early breast cancer detection. The physical basis for microwave detection is the significant contrast in dielectric properties between normal tissue and cancerous tissue [3]. As a result, the tumor microwave scattering cross-section is much larger than that of a normal breast tissue with the same size. The UWB microwave imaging approach transmits broadband microwave pulses from different locations on the breast surface and records the backscattered responses from the breast. The backscattered responses are processed coherently to form the image.

Previously, data-independent beamforming methods, such as the delay-and-sum (DAS) [4] and the microwave imaging via space-time (MIST) beamforming [5], have been used for MWI breast cancer detection in two-dimensional (2-D) and simplified three-dimensional (3-D) breast models. These works demonstrated the potential of using UWB microwave imaging for the detection of breast cancer. However, these data-independent methods have low resolution and poor interference suppression capability.

In this paper, we present two data-adaptive algorithms for microwave imaging, referred to as the robust weighted Capon beamformer (RWCB) method and the amplitude and phase estimation (APES) method. These data-adaptive methods outperform their data-independent counterparts in terms of higher resolution and better interference suppression capability. The weighted Capon beamformer (WCB) is a method similar to the standard Capon beamformer (SCB), but a weighting strategy is used to improve the beamformer performance. To make WCB robust, we adopt the main idea of robust Capon beamformer (RCB) [6] into WCB, and the resulting algorithm is referred to as the robust weighted Capon beamformer (RWCB). APES is derived based on the least squares fitting of the beamformer output [7] under the assumption that the signal waveform is known.

Data Model and Problem Formulation

Data collection and early-time response removal: We consider herein a bistatic radar model for the imaging system. A pair of transmitter/receiver antennas are used to scan the breast at different positions. During each scan, the antenna pair is located on the breast skin at a chosen position $\boldsymbol{r}_i = [x_i\ y_i\ z_i]^T$. Here, $(\cdot)^T$ denotes the transpose. A broadband microwave pulse is sent by the transmitter, and the backscattered signal is sampled by the receiver. Let $E_i(t), i = 1, \cdots, M$, denote the received signal by the i^{th} channel at time instant t, and let $\mathbf{r}_{iT}$ and $\mathbf{r}_{iR}$ denote the positions of the transmitter and receiver antennas for the i^{th} channel, respectively, where M is the number of channels or antenna pair positions. Our goal is to detect the tumor by constructing 3-D images of the backscattered energy $p(\boldsymbol{r})$ as a function of imaging location $\mathbf{r}$ within the breast.

There are early-time and late-time contents in the received backscattered signals. The early-time content is dominated by the incident pulse and reflections from the breast skin. The late-time content contains tumor backscattered signals and other backscattering due to the inhomogeneous fatty tissue, glandular tissue, and chest wall. Due to the small distances between the antennas and between the antenna pair and the skin, the magnitude of the early-time content is much larger than that of the late-time content. We must remove the early-time response to enhance the tumor response. Because the distance between the transmitter and the receiver is fixed and the skin tissues are similar at different positions, the signals recorded at various antenna locations have similar direct propagations and skin reflections. Hence we can remove the early-time content by subtracting a fixed signal out from all channels. This calibration signal $\bar{E}(t)$ can be obtained simply by averaging the recorded signals at all channels. After $\bar{E}(t)$ is subtracted out from each channel, we have the preprocessed signal $X_i(t) = E_i(t) - \bar{E}(t), i = 1, \cdots, M$.

Signal time-shifting, windowing, and compensation: For the i^{th} channel, we align the return from a specific imaging location $\mathbf{r}$ with the returns from the same location for the other channels by time-shifting the signal $X_i(t)$ a number of samples $n_i(r)$. The discrete-time delay between the antennas and $\mathbf{r}$ can be calculated as $n_i(\boldsymbol{r}) = \lfloor \frac{1}{\Delta t}[\|\boldsymbol{r}_{iT} - \boldsymbol{r}\|/C + \|\boldsymbol{r}_{iR} - \boldsymbol{r}\|/C]\rfloor$, where $\lfloor x \rfloor$ stands for rounding to the greatest integer less than x, C is the velocity of microwave propagating in breast tissues, and Δt is the sampling interval, which is assumed to be sufficiently small. The time-shifted signal is denoted as $\check{X}_i(t, \boldsymbol{r}) = X_i(t + n_i(\boldsymbol{r})), t = -n_i(\boldsymbol{r}), \cdots, T - n_i(\boldsymbol{r})$, where T is the maximum time (rounded to the nearest multiples of the sampling interval) needed by microwave pulse to propagate from the transmitter to the far side of the skin or chest wall and back to the receiver.

After time-shifting, the backscattered signals from location $\mathbf{r}$ are aligned so that they all start approximately from time $t = 0$ for all channels. Next the aligned signals are time windowed to isolate the backscattered signals from location $\mathbf{r}$. The windowed signals are denoted by $\check{X}_i(t, \boldsymbol{r}), t = 0, \cdots, N-1$, where $N\Delta t$ is the approximate duration of the backscattered signal from location $\boldsymbol{r}$.

Propagation attenuation occurs when the microwave propagates within the breast. The attenuation of the tumor responses at various channels is different because the distances from the transmitter to the imaging position $\mathbf{r}$ and back to the receiver are different. Here we only compensate out the attenuation due to the propagation and ignore the lossy medium effect because the propagation attenuation is the dominant factor. For the i^{th} channel, the compensation factor is given by $K_i(\boldsymbol{r}) = \|\boldsymbol{r}_{iT} - \boldsymbol{r}\|^2 \cdot \|\boldsymbol{r}_{iR} - \boldsymbol{r}\|^2$, and the compensated signal can be calculated as $y_i(t, \boldsymbol{r}) = K_i(\boldsymbol{r}) \cdot \check{X}_i(t, \boldsymbol{r}), t = 0, \cdots, N-1$.

Data Model: Without loss of generality, we consider imaging at the generic location $\mathbf{r}$ only. For notational convenience, we drop the dependence of $y_i(t, \boldsymbol{r})$ on $\mathbf{r}$, and simply denote it as $y_i(t)$. Now we consider the signal $\mathbf{y}(t) = [y_1(t)\ y_2(t)\ \cdots\ y_M(t)]^T, t = 0, \cdots, N-1$. After preprocessing, each snapshot $\mathbf{y}(t)$ can be modeled as

$$\mathbf{y}(t) = \mathbf{a} \cdot s(t) + \mathbf{e}(t) \tag{1}$$

where $s(t)$ is the backscattered signal, $\mathbf{a}$ denotes the steering vector, and $\mathbf{e}(t) = [e_1(t)\ e_2(t)\ \cdots\ e_M(t)]^T (t = 0, \cdots, N-1)$ is a term comprising both interference and noise. Since $\mathbf{y}(t)$ was properly time-shifted and compensated for, the steering vector $\mathbf{a}$ is assumed to be $[1\ 1\ \cdots\ 1]^T$. The problem of interest then is to estimate the backscattered signal $s(t)$ from $\mathbf{y}(t)$.

Adaptive Microwave Imaging

Robust Weighted Capon Beamformer (RWCB): The standard Capon beamformer (SCB) [8] considers the following problem

$$\min_{\mathbf{w}} \mathbf{w}^T \hat{\mathbf{R}} \mathbf{w} \quad subject \quad to \quad \mathbf{w}^T \mathbf{a} = 1 \tag{2}$$

where $\mathbf{w}$ is the beamformer's vector, and $\hat{\mathbf{R}} \triangleq \frac{1}{N} \sum_{t=0}^{N-1} y(t) \cdot y^T(t)$ is the sample covariance matrix.

The weighted Capon beamformer (WCB) uses a simple least squares estimate of $s(t)$ as a weighting function: $h(t) = \mathbf{y}^T(t) \cdot \mathbf{a}/\|\mathbf{a}\|^2 = \frac{1}{M} \sum_{i=1}^{M} y_i(t)$. Then WCB is obtained by solving the following optimization problem

$$\min_{w} \mathbf{w}^T \hat{\hat{\mathbf{R}}} \mathbf{w} \quad subject \quad to \quad \mathbf{w}^T \mathbf{a} = 1 \tag{3}$$

where the weighted sample covariance matrix is defined as $\hat{\tilde{\mathbf{R}}} = \frac{1}{N}\sum_{t=0}^{N-1}\mathbf{y}(t)\mathbf{y}^T(t)\cdot h^2(t)$. The solution of (3) is given by $\hat{\mathbf{w}}_{WCB} = \hat{\tilde{\mathbf{R}}}^{-1}\mathbf{a}/\mathbf{a}^T\hat{\tilde{\mathbf{R}}}^{-1}\mathbf{a}$ and $\hat{s}_{WCB}(t) = \hat{\mathbf{w}}_{WCB}^T\cdot\mathbf{y}(t)$. Then the backscattered energy can be calculated as $p(\mathbf{r}) = \sum_{t=1}^{N}\hat{s}^2(t)$ which will be regarded as the backscattered energy from position $\boldsymbol{r}$.

Note that the only difference between SCB and WCB is in their sample covariance matrices. The weighted sample covariance matrix used by WCB has an intuitively appealing interpretation: if $h(t)$ is large, which indicates that the signal content of $y(t)$ is large, we give $y(t)$ more weight when we estimate the covariance matrix, and vice versa. By this weighting strategy, the so-obtained beamformer "focuses" on the snapshots where the estimated signal content is large.

WCB has better resolution and much better interference rejection capability than the data-independent beamformers. However, similar to SCB, it suffers from severe performance degradations when some of the underlying assumptions on the environment, sources, propagation, or sensor array are violated. To improve the performance of WCB in the presence of model errors, we assume that the true steering vector is $\hat{\mathbf{a}}$, which is a vector in the vicinity of $\mathbf{a}$, and that the only knowledge we have about $\hat{\mathbf{a}}$ is that $\|\hat{\mathbf{a}}-\mathbf{a}\|^2 \leq \epsilon$ where ϵ is a user parameter. We adopt the recently developed robust Capon beamforming (RCB) [6] approach to make WCB robust against the errors in $\mathbf{a}$. To make this paper self-contained, we summarize the main steps of the RWCB approach below.

Consider the theoretical covariance matrix used by WCB

$$\tilde{\mathbf{R}} = \alpha\cdot\mathbf{a}\mathbf{a}^T + \mathbf{Q} \tag{4}$$

where $\alpha \triangleq \frac{1}{N}\sum_{t=0}^{N-1}s^2(t)h^2(t)$ and $\mathbf{Q} \triangleq \frac{1}{N}\sum_{t=0}^{N-1}h^2(t)\cdot E[e(t)e^T(t)]$. Due to the potential errors, the signal term in $\hat{\tilde{\mathbf{R}}}$ is not well described by $\alpha\cdot\mathbf{a}\mathbf{a}^T$, but by $\alpha\cdot\hat{\mathbf{a}}\hat{\mathbf{a}}^T$[9].

First, we assume $\hat{\mathbf{a}}$ is given (the determination of $\hat{\mathbf{a}}$ will be discussed later on). Then the RWCB problem can be re-formulated as

$$\min_{\mathbf{w}}\mathbf{w}^T\hat{\tilde{\mathbf{R}}}\mathbf{w} \quad \textit{subject} \quad \textit{to} \quad \mathbf{w}^T\hat{\mathbf{a}} = 1 \tag{5}$$

which has the solution

$$\hat{\mathbf{W}}_{RWCB} = \frac{\hat{\tilde{\mathbf{R}}}^{-1}\hat{\mathbf{a}}}{\hat{\mathbf{a}}^T\hat{\tilde{\mathbf{R}}}^{-1}\hat{\mathbf{a}}} \tag{6}$$

Next, we determine $\hat{\mathbf{a}}$ via a covariance fitting approach [8]. Since $\hat{\mathbf{a}}$ is a vector in the vicinity of $\mathbf{a}$ such that $\alpha\cdot\hat{\mathbf{a}}\hat{\mathbf{a}}^T$ is a good fit to $\hat{\tilde{\mathbf{R}}}$, we determine $\hat{\mathbf{a}}$ as the solution to the following optimization problem

$$\begin{aligned}\max_{\alpha,\hat{\mathbf{a}}}\alpha \quad \textit{subject} \quad \textit{to} \quad & \hat{\tilde{\mathbf{R}}} - \alpha\hat{\mathbf{a}}\hat{\mathbf{a}}^T \geq 0 \\ & \|\hat{\mathbf{a}}-\mathbf{a}\|^2 \leq \epsilon\end{aligned} \tag{7}$$

Usually, ϵ is determined by the various errors discussed previously. In practice, ϵ can be chosen experimentally by considering all the errors together. The above optimization problem can be solved as described in [6].

Amplitude and Phase Estimation (APES): Previously, we have developed the RWCB method based on the assumptions that the signal waveform can be estimated and then used as a temporal weighting function. In this subsection we present the amplitude and phase estimation (APES) method which explicitly assumes that the signal waveform is known.

Consider the following data model:

$$\mathbf{y}(t) = \mathbf{a}\beta\bar{s}(t) + \mathbf{e}(t), \quad t = 0, \quad \cdots, \quad N-1 \tag{8}$$

where β is the unknown amplitude of the backscattered signal with waveform $\bar{s}(t), t = 0, \cdots, N-1$, assumed to be known. To avoid a scaling ambiguity, we let $\sum_{t=0}^{N-1}[\bar{s}(t)]^2 = 1$. The APES method considers the following problem

$$\min_{\beta,\mathbf{W}}\frac{1}{N}\sum_{t=0}^{N-1}[\mathbf{w}^T\mathbf{y}(t) - \beta\bar{s}(t)]^2 \quad \textit{subject} \quad \textit{to} \quad \mathbf{w}^T\mathbf{a} = 1 \tag{9}$$

Here, the beamformer output $\mathbf{w}^T\mathbf{y}(t)$ is required to be as close as possible (up to a scaling factor β) to the known signal waveform $\bar{s}(t)$. By design, the APES beamformer can suppress the noise and interference, and at the same time, protect the signal of interest by enforcing the equality constraint.

Let $\mathbf{g} = \frac{1}{N}\sum_{t=0}^{N-1}\mathbf{y}(t)\bar{s}(t)$. A straightforward calculation shows that the criterion function in (9) can be rewritten as

$$\frac{1}{N}\sum_{t=0}^{N-1}[\mathbf{w}^T\mathbf{y}(t) - \beta\bar{s}(t)]^2 = (\frac{\beta}{\sqrt{N}} - \sqrt{N}\mathbf{w}^T\mathbf{g})^2 + \mathbf{w}^T\hat{\mathbf{R}}\mathbf{w} - N(\mathbf{w}^T\mathbf{g})^2 \tag{10}$$

So the minimization of (10) with respect to β is given by $\hat{\beta} = N \cdot \mathbf{w}^T\mathbf{g}$. Insertion of $\hat{\beta}$ into (10) yields the following minimization problem for the determination of the APES beamformer

$$\min_{\mathbf{w}} \mathbf{w}^T\mathbf{Z}\mathbf{w} \quad \textit{subject} \quad \textit{to} \quad \mathbf{w}^T\mathbf{a} = 1 \tag{11}$$

where we have defined $\mathbf{Z} = \hat{\mathbf{R}} - N \cdot \mathbf{g}\mathbf{g}^T$. The solution to (11) is readily obtained as $\hat{\mathbf{w}}_{APES} = \mathbf{Z}^{-1}\mathbf{a}/\mathbf{a}^T\mathbf{Z}^{-1}\mathbf{a}$ and $\hat{\beta} = N \cdot \mathbf{a}^T\mathbf{Z}^{-1}\mathbf{g}/\mathbf{a}^T\mathbf{Z}^{-1}\mathbf{a}$. Then the backscattered energy is $\hat{\beta}^2$.

Since we know the transmitted pulse and the dielectric properties of the tumor, the waveform of the backscattered microwave from a small tumor can be calculated theoretically. For simplicity, in the numerical experiments of the next section we just choose the normalized early-time response as the backscattered signal waveform in our numerical examples.

Conclusion

In this paper we have presented two data-adaptive microwave imaging (MWI) methods for breast cancer detection, namely the RWCB and the APES methods. A complex 3-D breast model was also developed to compare the performances of these adaptive imaging algorithms. The proposed data-adaptive methods produce better imaging results than their data-independent counterparts. Compared with MIST and DAS, RWCB and APES are more robust against noise. When the tumor is small (4 mm-diameter) in size, only RWCB and APES can still detect the tumor, while MIST and DAS cannot.

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REFERENCES

1. Fear, E. C., P. M. Meaney and M. A. Stuchly, "Microwaves for Breast Cancer Detection," *IEEE Potentials*, Vol. 22, No. 1, 12-18, February-March 2003.
2. Meaney, P. M., M. W. Fanning, D. Li, S. P. Poplack and K. D. Paulsen, "A Clinical Prototype for Active Microwave Imaging of the Breast," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 48, No. 11, 1841-1853, November 2000.
3. Surowiec, A. J., S. S. Stuchly, J. R. Barr and A. Swarup, "Dielectric Properties of Breast Carcinoma and the Surrounding Tissues," *IEEE Transactions on Biomedical Engineering*, Vol. 35, No. 4, 257-263, April 1988.
4. Fear, E. C., X. Li, S. C. Hagness and M. A. tuchly, "Confocal Microwave Imaging for Breast Cancer Detection: Localization of Tumors in Three Dimensions," *IEEE Transactions on Biomedical Engineering*, Vol. 49, No. 8, 812-822, August 2002.
5. Davis, S. K., E. J. Bond, X. Li, S. C. Hagness and B. D. Van Veen, "Microwave Imaging via Space-sime Beamforming for Early Detection of Breast Cancer: Beamformer Design in the Frequency Domain," *Journal of Electromagnetic Waves and Applications*, Vol. 17, No. 2, 357-381, 2003.
6. Li, J., P. Stoica and Z. Wang, "On Robust Capon Beamforming and Diagonal Loading," *IEEE Transactions on Signal Processing*, Vol. 51, No. 7, 1702-1715, July 2003.
7. Stoica, P., H. Li and J. Li, "A New Derivation of the APES Filter," *IEEE Signal Processing Letters*, Vol. 6, No. 8, 205-206, August 1999.
8. Stoica, P. and R. L. Moses, *Spectral Analysis of Signals, Prentice-Hall*, Englewood Cliffs, NJ, 2005, forthcoming.
9. Li, J. and P. Stoica, eds, *Robust Adaptive Beamforming*, John Wiley & Sons, New York, NY, 2005.